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Phenomenological Constraints on $\bar{\Lambda}$ and λ_1

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Abstract

Combining the experimental data on the inclusive decays $D \to X\,e\,\nu$, $B \to X\,e\,\nu$ and $B \to X\,\tau\,\nu$, we find severe constraints on the $\bar{\Lambda}$ and λ_1 parameters of the Heavy Quark Effective Theory. In particular, we get $\bar{\Lambda} < 0.7\,\mathrm{GeV}$. Our constraints further imply $m_c \geq 1.43\,\mathrm{GeV}$ and $m_b \geq 4.66\,\mathrm{GeV}$. We discuss future prospects of improving these bounds and their phenomenological implications.

The Heavy Quark Effective Theory (HQET) provides a systematic way of calculating $1/m_Q$ corrections to the model independent results of heavy quark symmetry [1]. For inclusive semileptonic B and D decay rates, Chay *et al.* [2] showed that the heavy quark limit ($m_Q \to \infty$) coincides with the free quark decay model and that there are no corrections to this result at order $1/m_Q$. The $1/m_Q^2$ corrections can be computed in terms of only two additional nonperturbative parameters [3].

HQET provides much more precise predictions for inclusive semileptonic decays than the spectator quark model [4]. One ingredient in the improvement is the incorporation of the nonperturbative $1/m_Q^2$ corrections which are, however, numerically quite small. More important is that HQET gives an unambiguous definition of the quark masses [5], and these masses determine the inclusive decay rates. Using the relation between the masses of heavy quarks in terms of the HQET parameters $\bar{\Lambda}$, λ_1 and λ_2 (see their definitions below) leads to a significant reduction in the theoretical uncertainties [4].

These HQET parameters are genuinely nonperturbative: at present they can only be estimated in models of QCD. In ref. [4] we used such theoretical ranges for the various parameters to calculate inclusive semileptonic B decay rates. Here we take the opposite approach and investigate what restrictions can be derived from the existing experimental data on these parameters. Our constraints help to pin down the physical values of the HQET parameters. This will allow testing various model calculations and will lead to significantly reduced theoretical errors in the predictions for inclusive heavy hadron decays.

We now turn to an explicit presentation of the relevant HQET parameters. In the framework of HQET, semileptonic decay rates depend on three parameters (besides the Fermi constant, CKM angles, and the well-known masses of physical particles): $\bar{\Lambda}$, λ_1, and λ_2. $\bar{\Lambda}$ is given by the matrix element [5]

$$\bar{\Lambda} = \frac{\langle 0|\bar{q}\, iv\cdot \overleftarrow{D}\, \Gamma\, h_v|M(v)\rangle}{\langle 0|\bar{q}\, \Gamma\, h_v|M(v)\rangle}\,, \tag{1}$$

where $M(v)$ denotes the meson state and h_v denotes the quark field of the effective theory with velocity v. This parameter describes, at leading order in $1/m_Q$, the mass difference between a heavy meson and the heavy quark that it contains, and sets the scale of the $1/m_Q$ expansion. The dimensionful constants λ_1 and λ_2 [6] parameterize the matrix elements of the kinetic and chromomagnetic operators, respectively, which appear in the Lagrangian of HQET at order $1/m_Q$:

$$\lambda_1 = \frac{1}{2}\langle M(v)|\bar{h}_v (iD)^2 h_v|M(v)\rangle\,, \tag{2a}$$

$$\lambda_2 = \frac{1}{2d_M}\langle M(v)|\frac{g_s}{2}\bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v|M(v)\rangle\,, \tag{2b}$$

where $d_P = 3$ and $d_V = -1$ for pseudoscalar and vector mesons, respectively. Whereas λ_1 is not renormalized due to reparameterization invariance [7], λ_2 depends on the scale.

The d_M factor appears because the chromomagnetic operator breaks the heavy quark spin symmetry. Consequently, the value of λ_2 can be easily extracted from the mass splitting between the vector and pseudoscalar mesons:

$$\lambda_2 = \frac{m_{B^*}^2 - m_B^2}{4} \simeq 0.12\,\mathrm{GeV}^2\,. \tag{3}$$

We expect this value of λ_2 to be accurate to within 10%, as a result of the finite b quark mass and the experimental uncertainties. There is no similarly simple way to determine λ_1 and $\bar\Lambda$.

The HQET parameters appear in the expansion of the heavy meson masses in terms of the charm and bottom quark masses:

$$m_B = m_b + \bar\Lambda - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \dots\,, \tag{4a}$$

$$m_D = m_c + \bar\Lambda - \frac{\lambda_1 + 3\lambda_2}{2m_c} + \dots\,. \tag{4b}$$

For each set of values $\{\bar\Lambda, \lambda_1, \lambda_2\}$, eqs.(4) determine m_c and m_b. The consistency of the heavy quark expansion requires that these values of the quark masses are used in the theoretical expressions for the decay rates. Then, a precise knowledge of $\bar\Lambda$ and λ_1 would allow accurate predictions for heavy hadron decays.

It was noticed in ref. [8] that semileptonic D decays provide a lower bound on the parameter λ_1 at each particular value of $\bar\Lambda$. At present, there exist three measurements of inclusive rates that provide bounds on $\bar\Lambda$ and λ_1: $D \to X\, e\, \nu$, $B \to X\, e\, \nu$, and $B \to X\, \tau\, \nu$. In what follows we discuss the implications of each of these processes in turn.

For $D \to X\, e\, \nu$, following ref. [8] (with minor modifications explained below), we compute the *rate* $\Gamma(D \to X\, e\, \nu)$ including order $1/m_c^2$ and order α_s corrections, and compare it to the experimental value of $BR\,(D \to X\, e\, \nu)/\tau_D$. We use [9]

$$BR\,(D^\pm \to X\, e^\pm\, \nu) = 17.2 \pm 1.9\,\%\,, \qquad \tau(D^\pm) = 1.066 \pm 0.023\,\mathrm{ps}\,. \tag{5}$$

The theoretical expression for the decay rate can be found in ref. [8]. Demanding that the theoretical prediction be compatible with the experimental result, we constrain the allowed range in the $\bar\Lambda - \lambda_1$ plane. This is plotted in fig.1. The allowed region is the area above the curve marked with (1). A large uncertainty arises in the calculation because the $\mathcal{O}(\alpha_s^2)$ corrections are unknown. Effectively, these corrections set the scale at which the $\mathcal{O}(\alpha_s)$ corrections should be evaluated. The curve in the figure results from the rather safe assumption that the scale is not above m_c. If one assumes that the scale is larger than some value, say $m_c/3$, then an upper bound on λ_1 can also be obtained. Such a bound does not play an essential role in our analysis, so we do not present it here. The interested reader may consult ref. [8]. In deriving our bound we took a more conservative approach than ref. [8]. We take exactly the same ranges for the various input parameters (except for α_s), but allow each of them to independently vary within their respective 1σ bounds. To indicate the difference between the bounds derived in these two different ways, assuming $\bar\Lambda > 0.237\,\mathrm{GeV}$ ref. [8] quotes $\lambda_1 > -0.5\,\mathrm{GeV}^2$ while we obtain $\lambda_1 > -0.65\,\mathrm{GeV}^2$. Our bound is likely to strengthen once $\mathcal{O}(\alpha_s^2)$ corrections are calculated or if the experimental lower bound on the branching ratio increases.

The decay $B \to X\, e\, \nu$ is analyzed in a similar way, namely we calculate the rate $\Gamma(B \to X\, e\, \nu)$ including order $1/m_b^2$, $1/m_c^2$ and α_s corrections, and compare it to the experimental value of $BR\,(B \to X\, e\, \nu)/\tau_B$. We use for the branching ratio [9]

$$BR\,(B \to X\, e\, \nu) = 10.7 \pm 0.5\,\%\,, \tag{6}$$

where we combined the systematic and statistical errors in quadrature, and for $\tau_B |V_{cb}|^2$ the most recent model independent determination from exclusive $B \to D^* \ell\, \nu$ decays [10],

$$\sqrt{\frac{\tau_B}{1.49\mathrm{ps}}}\, |V_{cb}| = 0.037 \pm 0.007\,. \tag{7}$$

This procedure yields the upper bound on $\bar\Lambda$ corresponding to the curve marked with (2) in fig.1. The effect of $\mathcal{O}(\alpha_s^2)$ corrections is likely to strengthen our bounds (by lowering the effective scale for α_s) but is expected to be small. The sensitivity to higher order corrections in the relations (4) is large. In deriving this bound we included estimates of these uncertainties. If the upper bound on $|V_{cb}|$ becomes lower, that would make this bound significantly stronger. For example, $\sqrt{\frac{\tau_B}{1.49\mathrm{ps}}}\, |V_{cb}| \leq 0.040$ would yield the upper bound given by the solid curve in fig. 1. Similarly, an improvement in the lower bound on the branching ratio would strengthen our bound.

The decay $B \to X\, \tau\, \nu$ has been investigated in detail in ref. [4], where the theoretical expression for the branching ratio can be found, including order α_s and $1/m_b^2$ corrections. This decay rate, normalized to the experimentally measured rate for $B \to X\, e\, \nu$, is a very reliable way of setting bounds on $\bar\Lambda$ and λ_1. The sensitivity to higher order QCD corrections is minute, as the correction to the ratio of rates is much smaller than the correction to each of them separately [4]. In addition, the ratio is independent of the overall $m_b^5\, |V_{cb}|^2$ factor. Taking into account the various uncertainties in the same conservative approach described in ref. [4], and using the recent measurement [11] with its statistical and systematic errors combined in quadrature

$$BR\,(B \to X\, \tau\, \nu) = 2.76 \pm 0.63\,\%\,, \tag{8}$$

we obtain a bound excluding large values of both $\bar\Lambda$ and λ_1. This is the curve marked with (3) in fig.1. This bound could become much stronger if the lower bound on the B to τ branching ratio becomes stronger. To demonstrate that, we plot (dashed line in fig.1) the bound that would correspond to $\sigma/2$ of the above range, *i.e.* $BR\,(B \to X\, \tau\, \nu) \geq 2.44\%$.

Each of the three experimental results that we have discussed determines an allowed *band* in the $\bar\Lambda - \lambda_1$ plane. We have not presented the upper bound from $D \to X\, e\, \nu$ because of the theoretical uncertainties discussed above. As for the lower bounds from $B \to X\, e\, \nu$ and $B \to X\, \tau\, \nu$, at present they are too weak to be interesting (excluding only negative values of $\bar\Lambda$ and large negative values of λ_1). Therefore, we do not present them either.

We have also investigated the implications of existing data on the inclusive rare decays $B \to X_s\, \gamma$ and $B \to X_s\, \ell^+\, \ell^-$. These processes yield no useful constraints at present. It is interesting to note, however, that as the lower bound on m_t will increase and the upper bound on $BR\,(B \to X_s\, \gamma)$ will decrease, the resulting bounds will be almost parallel (in the $\bar\Lambda - \lambda_1$ plane) to the bound provided by $D \to X\, e\, \nu$ decays. For example, if experiments find $m_t \simeq 170\,\mathrm{GeV}$ and $BR\,(B \to X_s\, \gamma) \leq 3 \times 10^{-4}$, then we will obtain the upper bound given by the dash-dotted curve in fig. 1. This will be important as the theoretical uncertainty in this process [12] is much smaller than in $D \to X\, e\, \nu$ decays.

Our results are summarized in fig.1. We learn that the two parameters $\bar\Lambda$ and λ_1 are significantly constrained by existing experimental data. Our most important result, an upper bound on $\bar\Lambda$, can be easily read off from this figure:

$$\bar{\Lambda} < 0.7\,\text{GeV}\,. \tag{9}$$

To set an upper bound on λ_1, we would need to know the size of the $\mathcal{O}(\alpha_s^2)$ corrections to semileptonic D decays. Assuming that the relevant value of the strong coupling constant α_s in D decays is $\alpha_s(m_c)$, $\alpha_s \le 2\alpha_s(m_c)$, $\alpha_s \le 3\alpha_s(m_c)$, we obtain the bounds $\lambda_1 < 0.6\,\text{GeV}^2$, $\lambda_1 < 0.7\,\text{GeV}^2$, $\lambda_1 < 0.8\,\text{GeV}^2$, respectively. The bound $\lambda_1 \lesssim 0.8\,\text{GeV}^2$ can also be obtained by combining the QCD sum rules prediction for $\bar{\Lambda}$ with the experimental results for $BR\,(B \to X\,\tau\,\nu)$ [4].

To set a lower bound on λ_1, we need a lower bound on $\bar{\Lambda}$. We should mention a theoretical model-independent bound [13] $\bar{\Lambda} > 0.237$ GeV (plotted with a dotted line in fig. 1). However, in ref. [14] it is argued that this bound is not valid. If $\bar{\Lambda} > 0.237\,\text{GeV}$, we obtain $\lambda_1 > -0.65\,\text{GeV}^2$ (and also $\lambda_1 < 1.2\,\text{GeV}^2$). The QCD sum rule prediction $\bar{\Lambda} \ge 0.50\,\text{GeV}$ [1,15] yields $\lambda_1 \ge -0.10\,\text{GeV}^2$.

It is instructive to compare these bounds to various model calculations. Our upper bound on $\bar{\Lambda}$ is hardly weaker than the allowed range of this parameter predicted by QCD sum rules, $\bar{\Lambda} = 0.57 \pm 0.07\,\text{GeV}$ [1,15]. The QCD sum rules estimates of the parameter λ_1 varied strongly over the past few years [16]. These sum rules are rather unstable, thus their predictions are uncertain and have large errors. Recent progress in this direction [17], however, indicates that λ_1 is likely to be negative and small in magnitude, $-0.3 \lesssim \lambda_1 < 0\,\text{GeV}^2$ [18]. This is certainly allowed by our bounds, but then $\bar{\Lambda}$ is likely to be smaller than 0.5 GeV. A recent calculation of λ_1 in the ACCMM model results $\lambda_1 \simeq -0.08\,\text{GeV}^2$ [19].

These experimental constraints on $\bar{\Lambda}$ and λ_1 can be easily translated into bounds on the charm and bottom quark masses. We find that in the experimentally allowed region

$$m_c \ge 1.43\,\text{GeV}\,, \qquad m_b \ge 4.66\,\text{GeV}\,. \tag{10}$$

We do not quote upper bounds on the quark masses, both because of the uncertain status of the model independent lower bound on $\bar{\Lambda}$, and because such a bound would also depend on assumptions about the higher order QCD corrections to semileptonic D decays.

A precise knowledge of the bottom and charm quark masses (and of λ_1) is important, for example, for a model independent determination of $|V_{cb}|$ from inclusive semileptonic $B \to X_c\,\ell\,\nu$ decays. When phenomenological constraints, like those derived in this paper, or possible theoretical model-independent bounds on $\bar{\Lambda}$ and λ_1 will become more restrictive, inclusive $B \to X_c\,\ell\,\nu$ decays may provide an alternative determination of $|V_{cb}|$, comparable in accuracy to that from the zero recoil limit of exclusive $B \to D^{(*)}\ell\,\nu$ decays.

The bounds for the quark masses in eq. (10) have important consequences for $BR\,(B \to X\,e\,\nu)$. The theoretical result for the semileptonic *branching ratio* derived by calculating both the semileptonic and hadronic widths, seems to constitute a puzzle at present [20]: theory predicts $BR\,(B \to X\,e\,\nu) > 0.12$, in conflict with the experimental result (6). While we have nothing to add here to the ongoing discussion on the uncertainties in the calculation of the hadronic width, we would like to recall that a solution of the problem prefers low quark masses [21]. Thus, our lower bounds on m_c and m_b point into the direction of increasing this problem, rather than eliminating it.

To summarize our main results, we showed that a combination of experimental data on semileptonic heavy meson decays gives the upper bound $\bar{\Lambda} < 0.7\,\text{GeV}$ and lower bounds on the heavy quark masses: $m_c \ge 1.43\,\text{GeV}$ and $m_b \ge 4.66\,\text{GeV}$. Improved experimental results

or theoretical calculations of $\mathcal{O}(\alpha_s^2)$ corrections will make these constraints significantly stronger.

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FIGURES

FIG. 1. The allowed range in the $\bar{\Lambda}-\lambda_1$ plane. The bounds are depicted by solid lines shaded on their excluded sides: (1) from $D \to X\, e\, \nu$ decays; (2) from $B \to X\, e\, \nu$ decays; (3) from $B \to X\, \tau\, \nu$ decays. The solid line would be the upper bound if experiment finds $\sqrt{\frac{\tau_B}{1.49\mathrm{ps}}}\,|V_{cb}| \leq 0.040$. The dashed line would be the upper bound if experiment finds $BR\,(B \to X\, \tau\, \nu) \geq 2.44\%$. The dash-dotted curve would be the upper bound if experiment finds $BR\,(B \to X_s\, \gamma) \leq 3 \times 10^{-4}$ and $m_t \simeq 170\,\mathrm{GeV}$.